CPM Philosophy Statements

Assessing what students understand requires more than one method.

In a balanced program, skill development is based upon problem solving and beginning understanding.

Teachers are responsible for actively guiding, supporting and summarizing.

Change takes time, effort, and support.
Mastery takes time, effort, and support.

Teachers need to establish and maintain effective study teams.

Students must be actively involved in their learning.

CCSS Mathematical Practices

1. MAKE SENSE of problems and persevere in solving them.

2. REASON abstractly and quantitatively.

3. CONSTRUCT viable ARGUMENTS and CRITIQUE the reasoning of others.

4. MODEL with mathematics.

5. Use APPROPRIATE TOOLS strategically.

6. Attend to PRECISION.

7. Look for and make use of STRUCTURE.

8. Look for and EXPRESS REGULARITY in repeated reasoning.

Synthesis of research that supports the principles of the CPM Educational Program.

This research synthesis comes in two parts. The first part is for a general audience—people who want an introduction to how CPM was developed and a brief discussion of the reasons for the choices we made. The second part contains specific references to the educational literature for the involved teacher, administrator or parent.

Overview. The writer-developers of CPM began with the belief that the primary goal of teaching mathematics should be ***long-term knowledge***. If learning does not persist past the end of the chapter or the end of the year, in what sense has the student learned anything useful? So the question became, what are the most effective ways to foster long term learning? Ultimately, the program was built around three fundamental principles informed by both theory and practice.

1. **Initial learning of a concept is best supported by discussions within cooperative learning groups guided by a knowledgeable teacher.**

2. **Integration of knowledge is best supported by engagement of the learner with a wide array of problems around a core idea.**

3. **Long-term retention and transfer of knowledge is best supported by spaced practice or spiraling.**

These principles derived from research provided a philosophy of how children learn and how teaching should occur in an ideal classroom. Then books were written to make this philosophy concrete and teachers were provided support so that they could use the books effectively.

The major change was to shift the focus of the student activity from being ***told a method*** or approach to being ***asked to solve problems*** designed to develop the method. The problems are attacked both individually and as a group with ideas freely exchanged as students grapple with new ideas or extensions of old ideas with the teacher as the ultimate resource. The mathematics is the same—students learn how to factor polynomials, for example—but they emerge from the CPM program with a deeper understanding of the topic and a better appreciation of where it fits into the whole structure of mathematics. Clearly some skills need to be mastered and become automatic, but simply memorizing what to do in a specific situation without an understanding of the reasons why method works too often leads to quick forgetting and no real long-term learning.

The knowledgeable reader will observe that this philosophy does not allow a scattershot approach to learning mathematics (the deservedly castigated “mile wide and inch deep” approach). Concepts need to be carefully organized so that the core ideas can be thoroughly acquired while relevant applications in related areas can be understood in terms of these core concepts.

This is why a single year of a CPM textbook is organized around no more than seven core ideas and why some of these core ideas extend over more than one year, the idea of proportion being one example. Then each chapter addresses one of these ideas in depth, developing and reinforcing the others as necessary, but always in terms of a theme problem toward which the students are aiming. The book provides structured guidance during class time for the students to explore questions in study teams where they can work together and exploit each other's insights to gain understanding. The teacher is always circulating through the classroom to monitor, guide, and intervene as necessary in the discussions so that students do not lead each other astray. CPM is not about having students reinvent wheels; it is about giving students the pieces and showing them a picture of a wheel so they can figure out how to put them together themselves.

Part of each night's homework is designed to reinforce the new ideas learned during class and the remaining questions are selected to recall and practice concepts and skills that were developed in previous lessons and chapters. Homework is essential for the internalization and reinforcement of ideas. Solving problems that are like those studied weeks or months before not only helps maintain the previous knowledge, it helps integrate the old knowledge with the new.

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Synthesis of research on cooperative learning.

Summary

It is unusual in educational research to have such unanimity of findings—in both individualistic settings and randomized experiments. The fact that these results are consistent for a wide span of ages and a wide set of topics indicates that a fundamental learning principle must be involved: ***social interaction increases the ability to learn ideas*** and to integrate them well into their existing cognitive structures. The techniques for using collaborative learning groups can undoubtedly be improved, but their efficacy is not in doubt.

Introduction

In the 1970's and 1980's studies began on the effects of peer tutoring—that is, having older or more able students tutor within classrooms. As was to be expected, students receiving the tutoring gained significantly. What was less expected was that **students doing the tutoring gained even more**. See Dineen et. al. (1977), and Cohen et. al. (1982) for summaries of this research and also see Semb et. al. (1993) for evidence that tutoring fosters longer-term retention.

At the same time that peer tutoring was being studied, teachers and researchers began experimenting with universal classroom tutoring by conceiving of the classroom as consisting of smaller cooperative learning groups where every student would have a chance both to tutor and to be tutored. The teacher would introduce the topic as needed and provide on-the-spot assistance where necessary, but much of the learning could reasonably be expected to take place between students as they grappled with ideas and tried to explain to each other or listen to explanations.

The effects of various forms of classroom cooperative learning groups (or small-group learning or learning teams) have now been studied extensively for over 30 years. For thorough overviews of the research, the reader is directed to Sharan (1980), Davidson (1985), Qin et. al (1995), Slavin (1996) and Springer et. al (1999). More in-depth information are provided by Johnson & Johnson (1989) and Slavin (1990). Other articles of general interest are Webb (1982) and (1991), Yager et. al. (1986), Palincsar & Brown (1988), Dees (1991) and Davidson & Kroll (1991). The main result of all of these tens of thousands of hours of research is that ***cooperative learning is a more effective way than direct instruction for students of all ages to learn most concepts—and is especially effective for students learning non-linguistic concepts*** (Qin, op.cit.).

Span of effect—age

We mentioned above that cooperative learning groups learning has been found to have a positive effect in a broad variety of learning situations. Fuchs et. al. (2002) did an experimental study of 20 first-grade classrooms randomly assigning half of the rooms to peer tutoring in pairs for mathematics and found that that students "at all points along the achievement continuum benefited." Skon et. al. (1981) had earlier discovered much the same thing for first graders learning mathematics is a smaller study.

A small sample of eighth graders learning science were studied by Chi et. al. (1994) who found that simply explaining to yourself prompted greater learning. For the same age group Shachar and Sharan (1994) found in a study of nine junior-high history and geography classes that "students' achievement scores were higher in classes taught with Group Investigation as compared with those taught with the [traditional Whole Class] method."

In another investigation of high-school students, Nichols (1996) randomly divided 80 geometry students into three classes: one taught by traditional methods and two taught in cooperative learning groups. At the end of the year he found that "students in the cooperative treatment groups exhibited significantly greater gains than the control group in geometry achievement" and also were superior for several affective goals. Two classes of pre-calculus students were compared by Whicker et. al. (1997) who found students in the cooperative learning groups had "higher tests scores than students in the comparison group" who studied alone.

At the college level, Crouch and Mazur (2001) reported on ten years of teaching physics at Harvard University using Peer Instruction (PI) and found that PI "increased student mastery of both conceptual reasoning and quantitative problem solving." Treisman (1985) recorded the effectiveness of having Black students work in cooperative learning groups outside of class. Finally, Cavalier et. al. (1995) studied 274 engineering employees in a required class and found that "practice conducted in a cooperative manner had a significant effect on performance and group behaviors."

Span of effect—ability

A commonly voiced concern by parents of high-ability students is that cooperative learning groups will interfere with their own child's learning. Stevens & Slavin (1995) addressed this concern directly and concluded after a two-year study in elementary school that "gifted students in heterogeneous cooperative learning classes had significantly higher achievement than their peers in enrichment programs without cooperative learning." More recently, Carter et. al. (2003) investigated achievement gains of high-ability fifth-grade students in a science unit and found no significant differences in the achievement of high-achieving students regardless of who they were partnered with.

At the high-school level, Saleh et. al. (2005) looked at students randomly assigned to homogeneous or heterogeneous ability groups in a plant biology course and the researchers concluded that "low-ability students achieve more … in heterogeneous groups … whereas high-ability students show equally strong learning outcomes in homogeneous and heterogeneous groups."

Thus it appears that the achievement and learning of high-ability students is not hindered by being members of cooperative learning groups and may, in fact, be increased by the fact that they have the chance to act as tutors within the group.

Additional benefits of collaborative group work

Gillies has done a series of studies investigating the long-range impact on students who work in cooperative groups. She has found (Gillies, 2000) that first-grade "children who have been trained to cooperate … are able to demonstrate these behaviors in reconstituted groups without additional training a year later." She followed up these results in Gillies (2002) by showing that fifth-graders who had been trained in cooperative groups two years earlier were "more cooperative and helpful than their untrained peers." So the impact of the ability to cooperate in a group lasts well beyond the end of the year or the situation in which that learning occurred.

Promising initial results that need further verification

An interesting interpretation of the positive effect of group work on students in college physics courses comes from Gautreau & Novemsky (1997) whose interviews with students suggest "that 'second teaching' takes place in small groups following 'first teaching' by the instructor." This interpretation would be consistent with the tutoring literature and with the oft-stated dictum that you cannot really understand a topic until you teach it. In addition, if this interpretation holds, it would also support the work of Gillies (2004) who concludes that structured groups (where every student is assigned a role within the group for that class day) are more effective in promoting learning than unstructured groups.

Kramarski (2004) looked at a modified collaborative learning environment for 196 eighth-grade mathematics students and concluded that students who were exposed to metacognitive instruction within cooperative learning groups did better than students who had only metacognitive instruction or who only worked in cooperative learning groups.

In a study of elementary school children, Webb (1991) and in a later study in four seventh-grade classes, Webb & Mastergeorge (2003) categorized the different help-seeking behaviors that are useful for students working in cooperative groups. Specifically they list: asking for specific explanations; persistence; modification of help-seeking strategies; and application to the problem under consideration. They also listed important help-giving behaviors as "providing explanations with verbally-labeled numbers and continued explaining instead of resorting to descriptions of numerical procedures."

Synthesis of research on problem-based learning.

Summary.
Traditional instruction with its emphasis on telling does not work well for long-term retention of knowledge for most children. Studies of students studying mathematics or science from first grade through college show that students ***retain more knowledge*** when they are taught using problem-based learning (PBL) than when they are simply told what to do. Problems engage the mental energies of students and allow them to develop cognitive understanding in a way that is more effective in the long term than simply being told a rule or procedure. Some research indicates that being told rules before attempting to forge a personal understanding can even interfere with deeper learning.

Teaching for understanding.
Everyone seems to agree that mathematics students should understand what they do when they learn math. As Skemp (1986, especially Chapter 12) has pointed out, however, there are two very different views about what constitutes "understanding." The first, which he terms "instrumental understanding," seems implicit in most mathematics textbooks—a student can carry out known procedures or solve standard problems according to a memorized method. An example would be how to subtract with borrowing which most students can memorize, but too few understand ***why*** it works.

The second, deeper, kind of understanding, Skemp terms "relational understanding," and refers to the ability of the individual to see the relations among the different parts of knowledge. This type of knowledge is more robust since the learner will have a sense of how things <u>should</u> work and be able to repair his or her own memory missteps with an understanding of the big picture. It is not the goal of relational understanding to create learners who respond automatically in narrow situations, but rather learners who see several different options for solving a problem and can make a reasoned choice among the alternatives. [We prefer the terms "procedural knowledge" and "conceptual knowledge" as used by Rittle-Johnson et.al. (2001) for the same ideas.]

The distinction is, of course, not precise. Different kinds of knowledge are important at different times. Some facts need to be acquired automatically so that the interesting work can proceed. No middle-school student should have to think to compute 5 + 9 or 17 x 10. At the same time, a student should understand that 3 x 4 = 4 x 3 with a better reason than "that is what the rule says[1]." Advanced students may also want to know that a formula exists for tan(2x) and can look it up without bothering to spend time memorizing the formula itself.

What is most important, however, is that the learning persists beyond the end of the chapter and beyond the end of the year. **If the learning does not persist so that it can be used in the future and cannot be utilized outside of a homework set, in what sense have students learned anything?** The problem for most students with memorizing and then practicing procedures is that this kind learning does not persist. It must be integrated with conceptual knowledge for permanence.

[1] A formally taught student might cite the Commutative Law. Terminology aside, they still should know **why**.

CPM is emphatically on the side of long-term learning which of necessity implies both kinds of understanding/knowledge. The question is—what are the most effective ways to help students achieve real understanding?

Focus on students

The answer is both simple and true across all disciples: a successful program must focus its energies on ***what students learn*** rather than measuring what teachers teach or what topics appear in the textbook or how many standards are met on paper. No matter what else happens—if the students do not learn, the teaching is not successful.

So how do we get students to learn and understand? Gardner (1983, 2000) has documented different learning styles for students. Similarly, different goals need different methods and no one method is superior for all children and all topics and all cases. Every successful program needs a mix of the methods. No one can be asked to discover what the definition of a trapezoid is and no one can be told the concept of an unknown. The former is a matter of social convention while the latter is such a deep concept that words barely help. At different times, students need different opportunities and different topics require different methods and different time frames.

Unfortunately, there is a strong belief on the part of many educators that students only need to be told what to do and, if they are told properly, they will learn the fact/skill/concept. It certainly ***seems*** efficient at conveying knowledge. The trouble with this belief is that **it is not true** except at a very young age or for students for whom the goal is merely procedural knowledge.

The problem with telling

The problems with teaching by telling have been amply documented by many researchers in mathematics and science at all levels and for most types of students. Carpenter et.al. (1998) followed students in grades 1-3 for three years and found that "students who used invented strategies before they learned standard algorithms demonstrated better knowledge of base-ten number concepts and were more successful in extending their knowledge to new situations than were students who initially learned standard algorithms." Similar results were reported for students this age by Hiebert & Wearne (1996) and Cauley (1998).

For sixth graders, Hmelo et.al. (2000) found that science design activities, which allow deeper explorations of how systems work helped students "learn more than students receiving direct instruction." For eighth graders, Woodward (1994) reported that students who learned the reasons for earth science phenomena "had significantly better retention of facts and concepts and were superior in applying this knowledge in problem-solving exercises."

At the college level, in a survey article McDermott & Redish (1999) have demonstrated that college level physics students do not learn some very basic content by lecture and this result has been duplicated with thousands of students at many institutions ranging from very selective private institutions and large state universities down through high schools. The work of Crouch & Mazur (2001) reporting on ten years using Peer Instruction (an interactive method of teaching) for the introductory physics courses at Harvard shows "increased student mastery of both conceptual reasoning and quantitative problem solving upon implementing Peer Instruction."

The work of Capon and Kuhn (2004) with adult students contrasting the outcomes of problem-based learning vs lecture/discussion showed that 6 weeks after instruction the lecture group was superior for one concept and the two groups were equivalent for the other. After 12 weeks concept retention was equal, but the problem-based learning group was superior in being able to explain what they had learned.

These results are not only true in mathematics and the sciences. Cobb (1999) reports on a study of students learning English in the Sultanate of Oman who learned vocabulary in two ways: dictionary definitions or by constructing their own definitions using the tools of lexicographers. "After 12 weeks, both groups were equal in definitional knowledge of target words, but lexicography group students were more able to transfer their work knowledge to novel contexts."

Problem-Based Learning

At the same time that studies have demonstrated the failure of direct instruction for average students, other studies have shown the advantages of problem-based learning (PBL). Most of these studies have been done with gifted students in K-12 or with engineering or medical students. See, for example, Albanese & Mitchell (1993) for an extensive review of the medical literature on PBL, Prince (2004) for a briefer summary on its uses with engineering students and Dods (1997) or Gallagher & Stepien (1996) for studies about gifted children learning with PBL.

These results confirm what has long been believed, that something akin to problem-based learning is superior for learning, when it is appropriate for the students involved. These earlier studies focused on students of ability—gifted elementary students or students in rigorous college programs. The implicit assumption has been that only a small minority of students can benefit from such an approach.

In the past decade, however, studies have found that well-designed PBL courses can benefit most, if not all, students. Songer, Lee & Kam (2002) report on a study of 19 urban sixth-grade classes showing students in all classrooms made significant content and inquiry gains. Kahle, Meece & Scantlebury (2000) report on eight middle schools in Ohio showed that teachers who used PBL or a modified form of it for teaching science "positively influenced urban, African-American science achievement." The study of Marx et.al. (2004) on approximately 8000 middle school students in the Detroit public schools showed (1) statistically significant increases on test scores and (2) that the effect increased for each of the three years that students were in the program. At the college level, Hake (2002) reports on the pre/post-test gains for more than 6500 students in introductory physics classes demonstrating the large positive effect of interactive engagement.

In smaller studies, Schneider et. al (2002) report on 10th and 11th grade students enrolled in Problem-Based Science was significantly better than matched groups on the National Assessment of Educational Progress science items, while Gallagher & Stepien (1996) reported that gifted students in a PBL class acquired as much content as students in a traditionally-taught class and acquired additional skills as well. This last result was sharpened in two further studies. Dods (1997) reports that lecture tended to widen the coverage as compared to a PBL class for gifted students in biochemistry, but ***understanding and retention [were] promoted by PBL*** [emphasis added]. A similar result was reported by in the meta-analysis of studies by Dochy et.al. (2003) who concluded that "students in PBL gained slightly less knowledge, but remember[ed] more of the acquired knowledge."

What these research pieces show is that the goals of long-term learning are maximized by problem-based learning and that virtually all students can profit from this form of education. In particular, we need not restrict this superior form of learning to the academically elite. Thus CPM structures its lessons so that students are told as much as necessary for learning a topic, but assumes based on the research cited above that most of the learning—the quality learning—will take place while working on problems.

Synthesis of research on spaced practice

Summary

The "spacing effect" is an overwhelmingly well-documented phenomenon that shows that learning is improved when the learning time is interrupted, or *spaced*, rather than being continuous, or *massed*. The effect has been observed in babies, in children and adults in numerous studies. The effect has been observed in learning mathematics, statistics, physics, languages, and such widely diverse skills as aircraft recognition and kayak rolls. Roughly speaking, as long as there is some latent memory of earlier learning of a skill, delaying the reinforcement by spacing improves both transfer and long-term learning.

There appear to be two principal reasons why this knowledge is not more widely applied in the classroom. First, little of the research has been done by educators. Most of the work has been done in psychology laboratories or for workplace training. Second—and this reason is probably more important for its lack of use—using spaced practice for learning rather than massed ***slows down the initial learning at the same time that it improves long-term retention and transfer.*** Thus teachers often feel that when students struggle to learn a new skill it is a bad thing because knowledge is mastered more slowly. However, the research is overwhelmingly positive that certain kinds of carefully designed struggle are beneficial.

Spacing thus has three positive effects for learning mathematics:

- it helps students **learn better;**
- it helps students **remember longer;** and
- it helps students **transfer their knowledge more effectively.**

Introduction

In what follows, we will be using the term "skills" to include such different notions as acquisition of concepts, algorithms and rules. It should be clear that the acquisition of a concept with all of the transfer implications included is very different than the learning of a rule with the ability to apply it appropriately in a narrow realm. However, most of the literature either ignores concept acquisition entirely or subsumes it under rule acquisition so we will rarely make the distinction.

In the past 70 years, dozens of researchers from psychology, workplace training and education have validated the "spacing effect," that is, the observation that learning improves when the learning time is interrupted, or *spaced*, rather than being continuous, or *massed*. Researchers who study workplace training refer to "distributed practice" or "spaced practice" (as opposed to "massed practice") for the cause of the spacing effect as they seek methods of improving the effectiveness of training programs or workers. See Schmidt (1992) and Salas (2001) for good review articles about studies on distributed practice.

Psychologists have verified the phenomenon in babies as young as three months of age in one study (Roveecollier, 1995) and in numerous studies for school-age children up to adults and in areas as diverse as rolling kayaks (Smith, 1995), aircraft recognition (Goettl, 1996) and learning languages (Bahrick, 1987, 1993). Because the spacing effect appears in so many contexts, it appears as Raaijmakers (2003, p. 432) comments, "that **basic principles of learning and retention are involved**." No wonder, then, that Dempsey (1998) laments the minimal impact that this research has had on educational practice.

Discussion of literature on distributed practice

The major problem from our standpoint with the literature on the effectiveness of distributed practice is that few of the studies have been done in educational fields, and fewer still in classrooms. Far more of this research has been done in looking at effective ways to provide workplace training. The constraints of workplace training are, of course, very different from those of the classroom since most such training is done over a short time interval (a few days at most) and most of it is limited to training a single skill or a small complex of closely-related skills. In addition, a large fraction of the research papers done by psychologists deal with the acquisition of motor skills such as balancing on a beam or shooting free throws which calls their relevance into question. With these limitations in mind, Salas and Cannon-Bowers (2001) did a comprehensive review article of 79 papers and concluded that distributing practice provides an advantage in learning and retention in a wide set of knowledge, skills and attitudes.

The only article that deals directly with mathematics learning (Mayfield, 2002) has some serious limitations for our purposes. First, it deals with college-age students who are learning algebra, so they are likely to be students who failed to learn algebra in high school. Second, the authors' philosophy of learning is basically to drill to automaticity on a narrow rule (three sessions of 50 similar problems each) before investigating what kind of review works best. They conclude that cumulative practice, that is, having students practice solving a mixture of previously-learned different types of problems in the review periods is better than simply additional blocks of practice.

Other classroom studies about the effects of distributed practice include those of Grote (1995) who studied the effects of distributed practice on high-school physics students, Smith (1995) who studied college statistics students and Bahrick (1987, 1993) who studied distributed practice in learning languages. Regardless of the direct relevance to K-12 mathematics learners, each of these studies concludes the same thing: **distributed classroom practice improves learning** and Grote specifically found that it improves transfer.

The question thus is not "Can the learning of mathematics be improved by making use of distributed practice?" but rather: "What are the most effective ways to distribute the practice in order to improve learning mathematics?"

The research article that seems to have the most to say about this question for typical 7-12 students in typical classrooms deals with something completely different: college students learning the game of Go, the classical Japanese board strategy game (Schilling, 2003). It is a beautifully designed study that gave 90 students, working in groups of three, ten hours in five-hour blocks over two consecutive days to learn the game of Go. This research reports that these groups of students learn to play Go best when they are also required to learn a related game, Reversi. The authors see Reversi as related to Go because both games involve placing pieces and controlling territory. In particular, what the study showed is that students who spent all of their sessions learning Go were less well able to play the game than students who spent about half of their time playing Reversi. The need for a related game was evidenced by the fact that students who had enforced interspersed time playing the game Cribbage played at about the same level as students who only played Go.

Retention and Transfer of Knowledge

Schmidt & Bjork (1992) point out that "the goal of training in a real-world setting is, or should be, to support … (a) the level of performance in the long term and (b) the capability to transfer that training to related tasks and altered contexts." Fisher (1996) and his colleagues summarize their position as:

> "Schmidt & Bjork (1992) have argued that in many circumstances it is a mistake to focus solely on the speed with which a given method of training leads to the desired level of proficiency. In particular, they argue that in some cases the method which speeds learning in the short term may actually hinder learning in the long term. …Shea & Morgan (1979) found that subjects were much faster throughout training [on motor tasks] in blocked practice. Yet, when subjects were returned for retention testing 10 days after the end of training, **subjects trained using random practice performed much better** in random testing than did subjects trained using blocked practice. Furthermore, subjects trained using random practice performed significantly better in blocked testing than did subjects trained using blocked practice." [emphasis added]

Since long-term retention of mathematical knowledge and the capability to transfer learning to related tasks are goals of CPM, it is clear that spaced practice is extremely desirable. At the same time, it is here that the problem occurs: spacing practice ***improves long-term learning and transfer*** while it ***decreases immediate acquisition***.

This trade-off was found not only for motor skills (Shea, 1979), but for many other learning situations as well. Pashler (2003) and his colleagues found that for learning vocabulary the spacing effect increased the number of errors during the learning phase, but improved later retention. Similarly, in a 9-year longitudinal study about retaining foreign vocabulary words, Bahrick (1993) and his colleagues found that 13 retraining sessions spaced at eight-week intervals was about as effective as 26 retraining sessions spaced at two-week intervals. In their abstract, they state, "The longer intersession intervals slowed down acquisition slightly, but this disadvantage during training was offset by **substantially higher retention.**" Smith (1984) showed that spreading out a single 8-hour statistics class over four days was significantly more effective than presenting the material in one day.

Transfer of learning.
In their summary of the literature on transfer of learning of cognitive tasks, Gick and Holyoak (1987) concluded that some conditions that enhance facilitate performance during the training period have a negative effect on retention or related tasks. Following up on this research, Schroth (1997) found that two different methods of delaying immediate learning improved long-term retention. Hesketh (1997) dealt specifically with transfer with college undergraduates and concluded that "methods of training that maximize immediate outcomes may do so at the cost of the longer-term benefits of developing transferable skills."

A related effect was described by Chen & Mo (2004) who found that "Exposure to less variant problems led to faster initial learning, but narrower and fixed schemas (mental set), whereas exposure to variant procedures led to slower initial learning, but broader and more flexible schemas." (Flexible schemas allow students to tackle kinds of problems they have not seen before.)

Implications for textbook design in mathematics
The sources cited above make it clear that for students in regular classrooms long-term learning and the ability to transfer this learning is facilitated by distributed practice and by using different kinds of problems built around the same theme. Massed practice provides the illusion of immediate learning but fails to maintain its effectiveness even after a reasonably brief time. Finding the balance requires extensive classroom testing, and one should probably err on the tradition side of caution, but it is now obvious that the literature raises serious questions about whether the design of traditional textbooks handicaps most students who attempt to learn from them.