

Da Vinci Innovation Academy  
K-4 Math Jumpstart  
August 21, 2012

Agenda

- Welcome and introductions
- Get to know your cohort
- Toilet paper mummy investigation
- Cognitively Guided Instruction (CGI) overview
- Open vs. closed questions
- What are good questions?
- Why ask good questions? / Be Mindful About...
- Some examples of good questions
- How to create good questions
- Practice writing good questions
- Art of questioning
- Recommended resources

# WRAP THE MUMMY

Pam is thirteen today.

She is holding a party at which she plans to play the game 'Wrap the mummy.'

In this game, players try to completely cover themselves with toilet paper.

A roll of toilet paper contains 100 feet of paper, 4 inches wide.

Will one toilet roll be enough to wrap a person?

Describe your reasoning as fully as possible.

(You will need to estimate the average size of an adult person.)



# **COGNITIVELY GUIDED INSTRUCTION (CGI)**

*From Hoosain and Chance's article in Teaching Children Mathematics, May 2004*

CGI is a research-based, problem-solving approach to teaching mathematics. As the name implies, an understanding of the learner's thinking guides instruction. CGI is based on the premise that children come to school with a great deal of mathematical knowledge; the task of the teacher is to uncover this knowledge and build on it.

In CGI, the teacher presents students with mathematical word problems set in the context of their environment and allows students the freedom to create their own strategies for solving the problems using available resources. They can use physical, pictorial, or symbolic representations and are encouraged to explore different ways to solve a particular problem. The process of obtaining an answer is more important than the answer itself.

The CGI teacher is a facilitator of learning rather than a disseminator of knowledge. He or she avoids imposing adult thinking on students. Then teacher spends a lot of time questioning and listening to students' explanations in order to determine how they are thinking. Therefore, the teacher must possess good questioning skills.

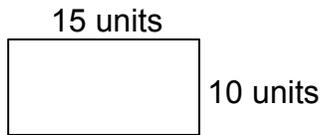
CGI places great emphasis on individualization, conceptual understanding, and higher-order thinking. More emphasis is placed on formative evaluation than on summative evaluation. The approach does not use any teaching materials such as textbooks, so the teacher must prepare his or her own problems. This responsibility makes teaching challenging for the CGI teacher.

# OPEN VS. CLOSED QUESTIONS

- **Closed questions** are those that simply require an answer or a response to be given from memory, such as a description of a situation or object or the reproduction of a skill.

Example:

What are the area and perimeter of the rectangle below?



- **Open questions** are those that require a student to think more deeply and to give a response that involves more than recalling a fact or reproducing a skill.

Example:

I want to make a garden in the shape of a rectangle. I have 30 meters of fence for my garden. What might be the area of the garden?

Most questions in mathematics lessons tend to be \_\_\_\_\_ questions.

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Identify each of the following as an open (O) or closed (C) question.

1. Circle the number that is even:      356                  843
2. The sum of the digits in a three-digit even number is 14. What might the number be?
3. What is 1,169 rounded to the nearest hundred?
4. A number has been rounded off to 1,200. What might the number be?
5. How many x's are in the next figure of the pattern below? Draw the figure.

x x x

x x x  
x x x x

x x x  
x x x x  
x x x x x

# THREE MAIN FEATURES OF “GOOD” QUESTIONS

## 1. They require more than remembering a fact or reproducing a skill.

Which example requires more than reproducing a skill?

- What is the average of 6, 7, 5, 8, and 4?
- The average of five numbers is 6. What might the numbers be?

## 2. They empower students to unravel their misconceptions.

Example:

- What number would you put in the blank to make this a true statement?

$$8 + 4 = \underline{\quad\quad} + 5$$

## 3. There may be several acceptable answers or several approaches.

Examples:

- In my pocket I have 75 cents. What coins might I have?
- Lucas has 14 packs of baseball cards. There are 11 cards in each pack. How many cards does Lucas have? Show your solution in more than one way.

## **WHY ASK GOOD QUESTIONS?**

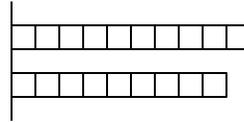
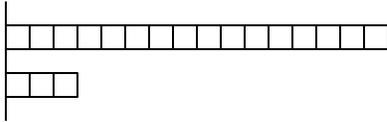
- To guide instruction cognitively
- To make learning active rather than passive
- To stimulate thinking and reasoning
- To be college- and career-ready
- To prepare for the Smarter Balanced Assessment

## **BE MINDFUL ABOUT...**

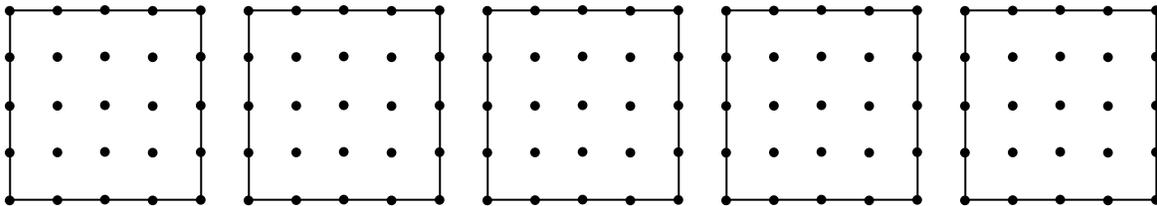
- Providing ample wait time
- Avoiding learned helplessness
- Creating a safe learning environment
- Applying the art of questioning
- Teaching like the Japanese

# SOME EXAMPLES OF GOOD QUESTIONS

1. The answer is 5. What is the question?
2. I bought something and paid for it with three coins. What might it have been and how much did it cost?
3. What might these graphs be about? Why do you think that?



4. Show different ways to divide each square into fourths.



5. Two-fifths of the students in a school borrow books from the library each day. How many students might there be in the school and how many of them borrow books each day?
6. What lengths could you make by combining these strips in different ways?

2"

3"

2"

4"

1"

7. About how much is a line of pennies that is 1 yard long worth? 1 mile long?
8. What are some box-shaped buildings that can be built with exactly 24 cubes?

# HOW TO CREATE GOOD QUESTIONS

Plan the questions in advance, as creating them is not something that can be done on your feet!

## **Step 1: Identify a topic.**

Example: Working with money

## **Step 2: Think of a closed question.**

Example:

An eraser costs 20 cents. If I give the cashier 25 cents, how much change did I get back?

OR

I bought an eraser for 20 cents. If I got 5 cents change, how much did money did I give to pay for it?

## **Step 3: Adapt it to make a good question.**

Example:

I bought something and got 5 cents change. How much did it cost and how much money did I give to pay for it?

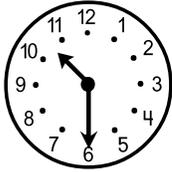
# PRACTICE WRITING GOOD QUESTIONS

Turn each closed question below into a good question.

1. What is the total value of the coins below?

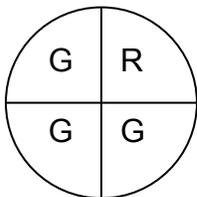


2. What time is shown on the clock below?



3.  $28 + 35 = \underline{\hspace{2cm}}$

4. What is the probability of spinning the color red?



# STANDARDS FOR MATHEMATICAL PRACTICE

*From the Common Core State Standards for Mathematics (CCSS\_M)*

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

## 1. Make sense of problems and persevere in solving them.

Example: Read and understand the problem or task at hand. Use concrete objects and pictorial representations. “Does my answer make sense?”

## 2. Reason abstractly and quantitatively.

Example: “There are 8 bags of cookies with the same amount of cookies in each bag. If there are 48 cookies how many cookies are in each bag?” Translate the situation into the equation:  $8 * \underline{\quad} = 48$  or  $48 / 8 = \underline{\quad}$  and then solve.

## 3. Construct viable arguments and critique the reasoning of others.

Example: Use more than one strategy to solve  $74 - 18$  and cite similarities and differences between strategies.

## 4. Model with mathematics.

Example: Draw pictures, make a t-table, a graph, and write an equation to answer the following: There are 5 petals on each flower. How many petals are on 10 flowers?

## 5. Use appropriate tools strategically.

Example: Use grid paper to draw all rectangles with an area of 12 square units.

## 6. Attend to precision.

Example: Measure objects correctly with a ruler and specify the units used.

## 7. Look for and make use of structure.

Example: 10, 20, 30, 40,     ,     ,     

## 8. Look for and express regularity in repeated reasoning.

Example: Discover the additive identity by examining patterns such as

$$3 + 0 = 3, 5 + 0 = 5, 18 + 0 = 18, \text{ so } a + 0 = \underline{\hspace{2cm}}$$

# THE ART OF QUESTIONING IN MATHEMATICS

*From The NCTM Professional Teaching Standards*

## **HELP STUDENTS TO RELY MORE ON THEMSELVES TO DETERMINE WHETHER SOMETHING IS MATHEMATICALLY CORRECT**

- “Why do you think that?”
- “Why is that true?”
- “How did you reach that conclusion?”
- “Does that make sense?”
- “Can you make a model and show that?”

## **HELP STUDENTS TO LEARN TO REASON MATHEMATICALLY**

- “Does that always work? Why or why not?”
- “Is that true for all cases? Explain?”
- “Can you think of a counter example?”
- “How could you prove that?”
- “What assumptions are you making?”

## **HELP STUDENTS LEARN TO ANALYZE, INVENT, AND SOLVE PROBLEMS**

- “What would happen if \_\_\_\_\_? What if not?”
- “Do you see a pattern? Explain.”
- “What are some possibilities here?”
- “Can you predict the next one? What about the last one?”
- “How did you think about the problem?”
- “What is alike and what is different about your two strategies?”

## **HELP STUDENTS CONNECT MATHEMATICAL IDEAS AND APPLICATIONS**

- “How does this relate to \_\_\_\_\_?”
- “What ideas that we have learned before were useful in solving this problem?”
- “Have we ever solved a problem like this one before?”
- “Can you give me an example of \_\_\_\_\_?”



<http://www.incrediblethings.com/home/giant-gumball-machine/>



<http://blog.popflys.com/2011/12/24/car-atlas/>

# RECOMMENDED RESOURCES

## Books

Carpenter, T. et. al. 1999. Children's Mathematics: Cognitively Guided Instruction. Heinemann. Portsmouth, NH

Small, Marian. 2012. Good Questions: Great Ways to Differentiate Mathematics Instruction, Second Edition. Teacher's College Press. New York, NY

Sullivan, Peter and Pat Lilburn. 2002. Good questions for math teaching: why ask them and what to ask [K-6]. Math Solutions Publications. Sausalito, CA

## Websites

Common Core State Standards for Mathematics  
<http://www.corestandards.org/assets/ccssi-introduction.pdf>.

Illustrative Mathematics Project  
<http://illustrativemathematics.org/standards/k8>

YouTube video: What would Khan Academy look like if it was made in Japan?  
<http://www.youtube.com/watch?v=CHoXRvGTtAQ>