COGNITIVELY GUIDED INSTRUCTION

An Introduction

DVIA Parent Educator Day
September 24, 2014

Nick Johnson

Cognitively Guided Instruction

Guiding principle:

- Children bring intuitive knowledge of mathematics to problem solving situations

Teaching approach:

- Elicit children's mathematical thinking and use it as the basis for instructional decisions
- Connect new mathematics ideas to prior knowledge to promote learning math with conceptual understanding

Overarching goal:

- Foster students' development as problem solvers who bring skill and confidence to mathematics situations
- Develop "mathematically proficient" students

- Janelle has 7 trolls in her collection. How many more does she have to buy to have 11 trolls?

Details Matter!

- Janelle has 7 trolls in her collection. How many more does she have to buy to have 11 trolls?

- Janelle had 11 trolls. She lost 7 of them. How many trolls does she have now?

- Janelle has 11 trolls. Rachel has 7 trolls. How many more trolls does Janelle have?

- Jane has 7 trolls in her collection. For her birthday, her friends gave her some more trolls. Now she has 11. How many did her friends give her?

Common Core State Standards: Grade 1

Operations and Algebraic Thinking   1.OA

☐ **Understand and apply properties of operations and the relationship between addition and subtraction.**

3. Apply properties of operations as strategies to add and subtract

Keisha has 6 beads. How many more beads does she need to collect to have 13 beads altogether?

Common Core State Standards: Grade 1

Operations and Algebraic Thinking   1.OA

- **Understand and apply properties of operations and the relationship between addition and subtraction.**

    3. Apply properties of operations as strategies to add and subtract

One way to represent Krystal's thinking

- 6 + __ = 13

- (6 + 6) + 1 = 13

- 6 + (6 + 1) = 13

- *Associative Property of Addition*

Activities to elicit and build upon children's mathematical thinking:

- Word Problems

- Warm-Up activities
  - How many ways…
  - One does not belong
  - Quick Images

- Counting Activities
  - Counting Collections
  - Choral Counting (skip counting)

- True/False Number Sentences

- Mental Math

Goal: To get kids talking, to establish classroom communities where multiple ways of thinking are valued, to build number sense, to increase student participation, etc.
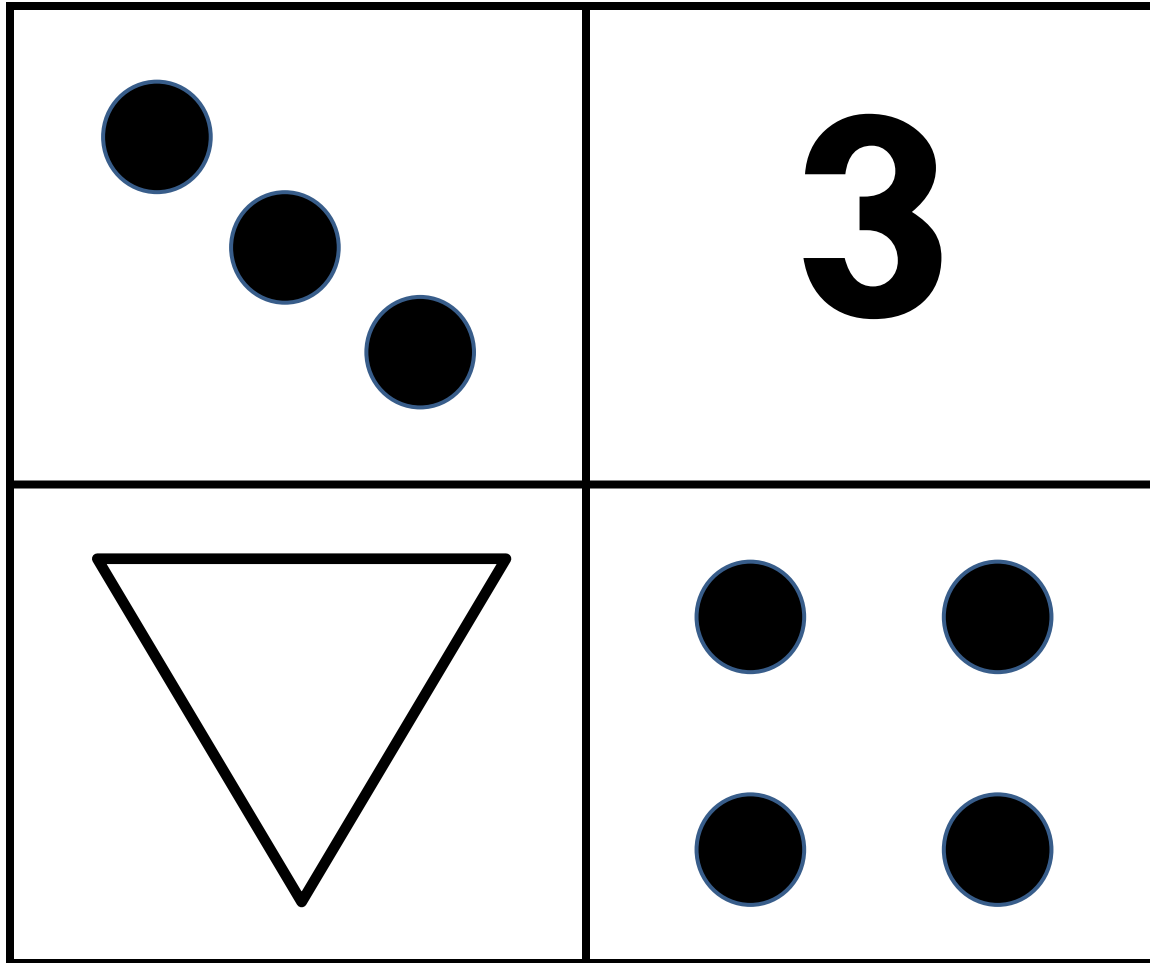
How many ways can you make…

450

One Does Not Belong

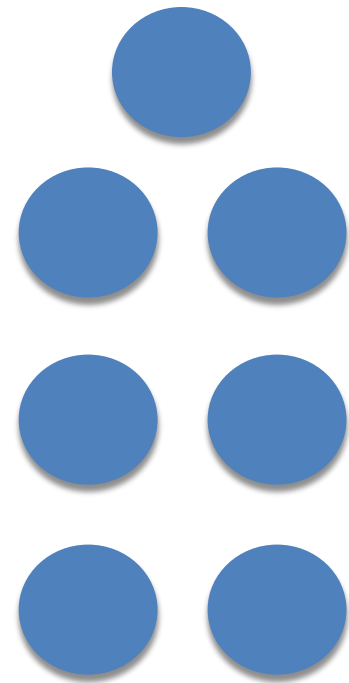|	
30	**36**
60	**360**

One Does Not Belong

One Does Not Belong

$.5$	$\dfrac{6}{8} - \dfrac{2}{8}$
$\dfrac{6}{12}$	$\dfrac{1}{4} \ x \ \dfrac{6}{3}$
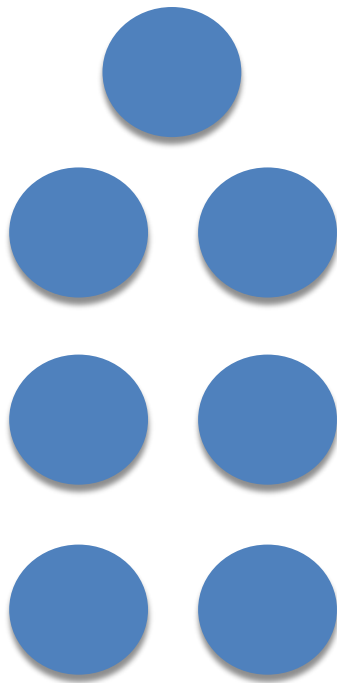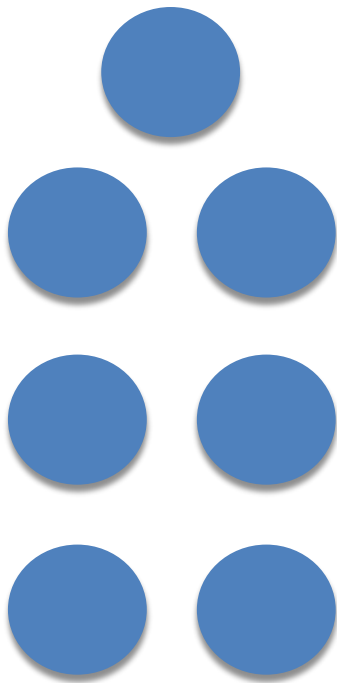
Quick Images

What can I do?

- Provide contexts for problem solving

- Ask "how did you figure that out?"

- Count… anything and everything!

http://talkingmathwithkids.com/

Problem solving

"What you do when you don't know what to do"

-Randy Philipp

Build a rich understanding of addition, subtraction, and place value (K-2)

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$

	Total Unknown	Addend Unknown	Both Addends Unknown[4]
Put Together/ Take Apart[5]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[6]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?, ? + 3 = 5$

Build a rich understanding of multiplication, division, and fractions (3-5)

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$ and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$ and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[8] Area[9]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

"The thesis of CGI is that children enter school with a great deal of informal or intuitive knowledge of mathematics that can serve as the basis for developing understanding of the mathematics of the primary school curriculum. Without formal or direct instruction on specific number facts, algorithms, or procedures, children can construct viable solutions to a variety of problems."

-*Children's Mathematics*, p. 4

PART 2

Cognitively Guided Instruction

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Mr. McClure had 18 balloons that he wanted to give to 4 kids. If he wants each kid to have the same number of balloons, how many balloons does each kid get?

Details Matter!

Julie (3rd grade)

Julie (again)

Mrs. Navarro has 24 pieces of candy. She wants to put 4 pieces in a bag. How many bags of candy can she make?

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Counting Collections

True/False number sentences:

- $8 + 4 = 7 + 5$

- $120 = 100 + 2 + 0$

- $147 + 298 - 298 = 147$

- $5 \times 62 = 10 \times 31$

- $9 - 7 = 7 - 9$

Counting & Number Patterns

4	8	12	16	20
24	28	32	36	40
44	48	52	56	60
64	68	72	76	80
84	88			

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